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Optical Theorem in Curved Space-Time Quantum Field Theory

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A structural analysis is given of the optical theorem in the S-matrix approach to mutually interacting quantum fields in classical Robertson-Walker universes. As a case study, the $\phi\psi^2$-interaction of conformally coupled massive (ϕ) and massless (ψ) Klein-Gordon particles is studied. Based on the outgoing massless particles as indicator configuration, the physical interpretation is reduced to the corresponding added-up probabilities. Several examples are discussed in an in-in scheme which has the advantage that only a few non-Minkowskian in-in Feynman diagrams are involved.

1. INTRODUCTION

A full quantum field theory of several mutually interacting quantum fields in given classical external gravitational fields is needed when studying high-energy processes in very early cosmic time. In several case studies (Audretsch and Spangehl, 1985, 1986, 1987; Audretsch *et al.*, 1987) we have discussed the question: How are Minkowskian cross sections and decay rates modified in an expanding universe? To do so, we have assumed cosmic expansion laws and mutual interactions which allow an exact treatment of the effects up to a certain order of the coupling parameter. See these articles for notions and conventions and for the underlying concept. For a survey of the literature see also the reviews in Birrell (1981), Birrell and Davies (1982), and Ford (1984).

In the following I try to enrich the calculation scheme in structurally discussing the optical theorem in cosmological universes. Because it has a direct formulation by in-in transition amplitudes, it fits very well into the

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in-in approach and can immediately be reduced to added-up transition probabilities. I will show this for several processes in detail.

As unquantized cosmological background I consider Robertson-Walker universes. They are conformally flat. I study the interaction

$$\mathscr{L}_I = \lambda f(x)\phi\psi^2 \tag{1.1}$$

[where $f(x)$ is some function of x and λ is the coupling parameter] between two types of neutral scalar particles described by the massive Klein-Gordon field ϕ and the massless field ψ. The corresponding field equations are assumed to be the massive and massless conformally coupled Klein-Gordon equations. Because of this choice, the massless particles realize the curved space-time only via the mutual interaction with the ϕ-particles. Furthermore, the ψ-particles will be represented in the calculations as in the Minkowski space essentially by plane waves.

I refer to the metric of the Robertson-Walker universe in its conformally flat form, the conformal time being η. The expansion law $a(\eta)$ may remain unspecified, but the in- and out-region ($\eta \to -\infty$, $\eta \to +\infty$) must allow the definition of particles.

The main consequence of the specifications above is that the energy is not conserved. This leads to important non-Minkowskian contributions to the effects. But there are conserved 3-momentum parameters $\mathbf{p}, \ldots, \mathbf{k}, \ldots$. The measured 3-momentum is $\mathbf{p}/a(\eta)$. Massive particles are created out of the vacuum also when the mutual interaction between the ϕ- and ψ-particles is switched off (zeroth-order process). There is no corresponding creation of massless ψ-particles out of the curved background.

The quantum field-theoretic situation is non-Minkowskian, because there are two different definitions of massive particles in the in- and out-region specified by the particular behavior of the Klein-Gordon solutions for $\eta \to -\infty$ and $\eta \to +\infty$, respectively. There are two complete sets of orthonormal solutions $\{u_\mathbf{p}^{\text{in}}\}$ and $\{u_\mathbf{p}^{\text{out}}\}$ describing ϕ-particle modes in the respective regions. Expand the massive field operator ϕ according to

$$\phi(x) = \sum_\mathbf{p} [a_\mathbf{p}^{\text{in}} u_\mathbf{p}^{\text{in}}(x) + a_\mathbf{p}^{\text{in}\dagger} u_\mathbf{p}^{\text{in}}(x)^*] \tag{1.2}$$

with operators satisfying the usual commutation relations

$$[a_\mathbf{p}^{\text{in}}, a_\mathbf{q}^{\text{in}\dagger}] = \delta_{\mathbf{pq}}, \qquad \text{rest} = 0 \tag{1.3}$$

The same is done for the out-region with regard to $\{u_\mathbf{p}^{\text{out}}\}$, leading to the operators $a_\mathbf{p}^{\text{out}}$.

For the massless ψ-particles the particle concepts in the in- and out-region are, because of the conformal invariance, based on the same set $\{v_\mathbf{p}(x)\}$ of solutions which are proportional to $\exp(i\mathbf{kx})$. The particle operators in the two asymptotic regions obtained from a decomposition of ψ agree,

$$b_{\mathbf{k}}^{\text{in}} = b_{\mathbf{k}}^{\text{out}} = b_{\mathbf{p}} \tag{1.4}$$

The states with a definite number of particles refer to one of the respective particle concepts. For the in-region we introduce the in-vacuum state

$$a_{\mathbf{p}}^{\text{in}}|0\ \text{in}\rangle = 0, \qquad b_{\mathbf{k}}^{\text{in}}|0\ \text{in}\rangle = 0 \qquad \forall \mathbf{p}, \mathbf{k} \tag{1.5}$$

and particle states according to

$$\begin{aligned} |e^{\phi} s^{\psi}\ \text{in}\rangle &= a_{\mathbf{p}}^{\text{in}\dagger} \ldots b_{\mathbf{k}}^{\text{in}\dagger} \ldots |0\ \text{in}\rangle \\ &= |1_{\mathbf{p}}^{\phi} \ldots 1_{\mathbf{k}}^{\psi} \ldots |\text{in}\rangle \end{aligned} \tag{1.6}$$

Corresponding definitions are made with reference to the out-region.

2. ADDED-UP TRANSITION PROBABILITIES AND INDICATOR CONFIGURATIONS

We are working in the interaction picture using an in–out scheme based on the S-matrix,

$$\begin{aligned} S &= \lim_{\varepsilon \to 0} \hat{T} \exp\left[i \int \mathscr{L}_I \exp(-\varepsilon|\eta|)\, d^4x \right] \\ &= 1 + iT = 1 + iT^{(1)} + iT^{(2)} + O(\lambda^3) \end{aligned} \tag{2.1}$$

$\hat{T}$ is the time-ordering operator; ε is the adiabatic switch-off parameter. The order with regard to λ is indicated in brackets.

As shown in Audretsch and Spangehl (1985), an in–out transition probability amplitude with states containing a finite number of particles loses mathematically its sense in our space-time. From the physical point of view this is a consequence of the fact that the gravitational background causes the creation of massive particles in all modes and that a particle counter for massive particles always registers the combined effect of background and mutual interaction with no possibility of discrimination. On the other hand, massless particles registered in the out-region have never come out of the background. They always go back to the mutual interaction and are therefore good indicators for the outcome of this interaction. If one wants to have statements regarding the mutual interaction, with a clear operational meaning, one must refer to the massless particles.

The concept of the added-up probability introduced in Audretsch and Spangehl (1985) is based on this. It starts with in-out amplitudes, but asks for the appearance of a particular indicator configuration in the out-region,

$$w^{\text{add}}(s^{\psi} \mid c^{\phi} r^{\psi}) = \sum_{d} |\langle \text{out } d^{\phi} s^{\psi} | S | c^{\phi} r^{\psi} \text{ in}\rangle|^2 \tag{2.2}$$

It contains a sum over all outgoing massive states and answers the question: What is the probability that a particular state of massless particles $|s^{\psi} \text{ out}\rangle$ will be found in the out-region regardless what has happened to the massive states? When there is no massless particle going out, we write $w^{\text{add}} = \langle 0^{\psi} | c^{\phi} r^{\psi} \rangle$.

As shown in Audretsch and Spangehl (1985), w^{add} can be reduced to in-in amplitudes

$$w^{\text{add}}(s^{\psi} \mid c^{\phi} r^{\psi}) = \sum_{d} |\langle \text{in } d^{\phi} s^{\psi} | S | c^{\phi} r^{\psi} \text{ in}\rangle|^2 \tag{2.3}$$

Characteristic for this and other in-in expressions is that they can be obtained in working out only a few in-in transition amplitudes. The second advantage is that this can be done in close analogy to the procedure everyone is used to in the Minkowskian situation. In-in Feynman diagrams can be constructed in the usual way. There is 3-momentum parameter conservation. But note that because of the lack of energy conservation, additional diagrams may appear. There is a Wick theorem for in-operators. The in-Feynman propagator for the ϕ-field is

$$\begin{aligned} i\Delta_{\text{F}}^{\text{in}}(x, y) &= \langle \text{in } 0 | \hat{T} \phi^{\text{in}}(x) \phi^{\text{in}}(y) | 0 \text{ in}\rangle \\ &= \underbrace{\phi^{\text{in}}(x) \phi^{\text{in}}(y)} \end{aligned} \tag{2.4}$$

When transcribing a diagram into integrals, one has to take into account that instead of the Minkowski-space plane waves, now the in-particle solutions $u_{\mathbf{p}}^{\text{in}}(x)$ and $v_{\mathbf{q}}^{\text{in}}$ have to be taken, where the latter is proportional to a plane wave.

Because the added-up probability w^{add} is a probability with direct operational meaning, I will base the subsequent physical discussion of the optical theorem on this concept and express terms which are to be interpreted by w^{add}. The starting point will always be the analysis of the contributing in-in Feynman diagrams.

3. THE OPTICAL THEOREM

Related to the completeness of the in- and out-states is the unitary of the S-matrix. We have

$$S^{\dagger} S = 1 \tag{3.1}$$

or, with (2.1),

$$T^\dagger T = -i(T - T^\dagger) \tag{3.2}$$

and therefore

$$\langle \text{in } a | T^\dagger T | a \text{ in}\rangle = 2 \,\text{Im}\langle \text{in } a | T | a \text{ in}\rangle \tag{3.3}$$

Inserting on the left-hand side the representation of the unity operator by in-states, we obtain the optical theorem which represents the restrictions implied by the unitarity of the S-matrix:

$$\sum_{e,s} |\langle \text{in } e^\phi s^\psi | T | d^\phi r^\psi \text{ in}\rangle|^2 = 2 \,\text{Im}\langle \text{in } d^\phi r^\psi | T | d^\phi r^\psi \text{ in}\rangle \tag{3.4}$$

Introducing added-up transition probabilities gives the form

$$\sum_{s} w^{\text{add}}(s^\psi \,|\, d^\phi r^\psi) = 2 \,\text{Im}\langle \text{in } d^\phi r^\psi | T | d^\phi r^\psi \text{ in}\rangle \tag{3.5}$$

The fact that the theorem can be formulated within the in–in formalism has once more the consequence that in a certain order of the coupling parameter the equation contains only a very limited number of terms. This will be demonstrated below.

The optical theorem relates the imaginary part of an in–in forward scattering amplitude with the total added-up transition probability into all massless states which can be reached according to the interaction. But in contrast to the situation in Minkowski space, we have no energy conservation. The sum on the left-hand side of (3.5) will therefore in general not degenerate to one term. For the same reason, the right-hand side of (3.5) may consist of more than one amplitude, because T will contain several terms according to Wick's theorem. This restricts the possibilities of a direct application of the optical theorem in curved space-time quantum field theory. In order to obtain expressions which can be applied to specific physical situations, one has to combine several results. I will demonstrate these characteristics of the optical theorem in the following examples.

4. TRANSITIONS FROM THE VACUUM

For the vacuum as in-going state the optical theorem (3.4) reduces, with

$$\sum_{e,s} |\langle \text{in } e^\phi s^\psi | T^{(1)} | 0 \text{ in}\rangle|^2$$
$$= \sum_{\mathbf{q},\mathbf{t}} |\langle \text{in } 1^\phi_{-\mathbf{q}} 1^\psi_{\mathbf{t}} 1^\psi_{\mathbf{q}-\mathbf{t}} | T^{(1)} | 0 \text{ in}\rangle|^2 + \sum_{\mathbf{q}} |\langle \text{in } 1^\phi_{\mathbf{q}} | T^{(1)} | 0 \text{ in}\rangle|^2 \tag{4.1}$$

to

$$2 \,\text{Im}\langle \text{in } 0 | T^{(2)} | 0 \text{ in}\rangle = \sum_{\mathbf{q},\mathbf{t}} |\langle 1\text{a}\rangle|^2 + \sum_{\mathbf{q}} |\langle 1\text{b}\rangle|^2 \tag{4.2}$$

(a) (b)

Fig. 1.

where $\langle 1a\rangle$ denotes the in–in matrix element obtained in working out the diagram of Figure 1a according to the in–in Feynman rules. The evaluations of the in–in Feynman diagrams of the other figures are denoted correspondingly.

With regard to a physical interpretation, we obtain that the imaginary part of the vacuum–vacuum amplitude in the in–in scheme is equal to the sum out of the total added-up amplitude for the transition of the vacuum into a state with two massless particles and of the transition into a state with no massless particles:

$$2\,\mathrm{Im}\langle \mathrm{in}\ 0|\,T^{(2)}|0\ \mathrm{in}\rangle=\sum_{\mathbf{q},\mathbf{t}} w^{\mathrm{add}}(1_{\mathbf{t}}^{\psi}1_{\mathbf{q}-\mathbf{t}}^{\psi}|\,0)^{(2)}+w^{\mathrm{add}}(0^{\psi}\,|\,0)^{(2)} \tag{4.3}$$

Note that according to the definition of the added-up probability, there may be many massive particles going out as the result of the mutual interaction. There are two vacuum–vacuum graphs contributing to the imaginary part (see Figure 2):

$$\mathrm{Im}\langle \mathrm{in}\ 0|\,T^{(2)}|0\ \mathrm{in}\rangle=\mathrm{Im}\langle 2a\rangle+\mathrm{Im}\langle 2b\rangle \tag{4.4}$$

A physically reasonable quantity which can in this context be worked out on the basis of the optical theorem is the total added-up probability for the decay of the vacuum into states with massless particles. Because of (4.3), this is given by

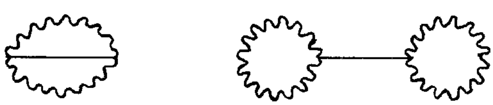
$$\sum_{\mathbf{q},\mathbf{t}} w^{\mathrm{add}}(1_{\mathbf{t}}^{\psi}1_{\mathbf{q}-\mathbf{t}}^{\psi}|0)^{(2)}=2\,\mathrm{Im}\langle \mathrm{in}\ 0|\,T^{(2)}|0\ \mathrm{in}\rangle-\sum_{\mathbf{q}}|\langle 1b\rangle|^{2} \tag{4.5}$$

Equation (4.4) will be useful in the following considerations.

5. DECAY OF A PAIR OF MASSLESS PARTICLES

The possible transitions according to the in–in scheme for the case of two massless particles going in are

$$\sum_{e,s}|\langle \mathrm{in}\ e^{\phi}s^{\psi}|\,T^{(1)}|1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}\ \mathrm{in}\rangle|^{2}=|\langle 3\rangle|^{2}+\sum_{\mathbf{q},\mathbf{t}}|\langle 1a\rangle|^{2}+\sum_{\mathbf{q}}|\langle 1b\rangle|^{2}+\sum_{\mathbf{k}}|\langle 5\rangle|^{2}+\sum_{\mathbf{k}\leftrightarrow\mathbf{l}}|\langle 5\rangle|^{2} \tag{5.1}$$



(a) (b)

Fig. 2.

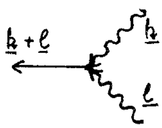


Fig. 3.

where reference is made to Figures 3 and 5. The amplitude $\langle 3\rangle$ is related to the probability that no massless particles reach the out-region:

$$w^{\mathrm{add}}(0^{\psi}\,|\,1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi})^{(2)} = |\langle 3\rangle|^2 \tag{5.2}$$

The matrix elements $\langle 1\mathrm{a}\rangle$, $\langle 1\mathrm{b}\rangle$ and $\langle 5\rangle$ have to be included in (5.1), because the possibility $\hat{w}$ that the two massless particles and the possibilities $\tilde{w}$ that one of the massless particles reappear in the out-region are contained in the sum

$$w^{\mathrm{add}}(1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}\,|\,1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi})^{(2)} = \sum_{\mathbf{q},\mathbf{t}} |\langle 1\mathrm{a}\rangle|^2 + \sum_{\mathbf{q}} |\langle 1\mathrm{b}\rangle|^2 \tag{5.3a}$$

$$\tilde{w}(1_{\mathbf{k}}^{\psi}\,|\,1_{\mathbf{k}}^{\psi})^{(2)} = \sum_{\mathbf{k}} |\langle 5\rangle|^2 \tag{5.3b}$$

Inserting this in (5.1), one finds that the optical theorem obtains in this case the operational interpretation

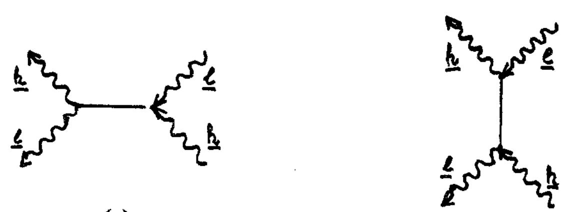
$$\begin{aligned} 2\,\mathrm{Im}\langle \mathrm{in}\ 1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}|\,T^{(2)}|1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}\ \mathrm{in}\rangle &= w^{\mathrm{add}}(0^{\psi}|1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi})^{(2)} + w^{\mathrm{add}}(1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}|1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi})^{(2)} \\ &\quad + \tilde{w}(1_{\mathbf{k}}^{\psi}|1_{\mathbf{k}}^{\psi})^{(2)} + \tilde{w}(1_{\mathbf{l}}^{\psi}|1_{\mathbf{l}}^{\psi})^{(2)} \end{aligned} \tag{5.4}$$

For the imaginary part we have

$$\begin{aligned} \mathrm{Im}\langle \mathrm{in}\ 1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}|\,T^{(2)}|1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi}\ \mathrm{in}\rangle &= \mathrm{Im}\langle \mathrm{in}\ 0|\,T^{(2)}|0\ \mathrm{in}\rangle \\ &\quad + \mathrm{Im}\langle 4\mathrm{a}\rangle + \mathrm{Im}\langle 4\mathrm{b}\rangle \\ &\quad + \mathrm{Im}\langle 6\mathrm{a}\rangle + \mathrm{Im}\langle 6\mathrm{b}\rangle \\ &\quad + \underset{\mathbf{l}\leftrightarrow\mathbf{k}}{\mathrm{Im}}\langle 6\mathrm{a}\rangle + \underset{\mathbf{l}\leftrightarrow\mathbf{k}}{\mathrm{Im}}\langle 6\mathrm{b}\rangle \end{aligned} \tag{5.5}$$

Equating (5.5) and (5.4), we obtain with (4.2), (5.3), (6.2) and (6.6) (see Figure 4)

$$w^{\mathrm{add}}(0^{\psi}\,|\,1_{\mathbf{k}}^{\psi}1_{\mathbf{l}}^{\psi})^{(2)} = 2\,\mathrm{Im}\langle 4\mathrm{a}\rangle + 2\,\mathrm{Im}\langle 4\mathrm{b}\rangle \tag{5.6}$$



(a) **(b)**

Fig. 4.

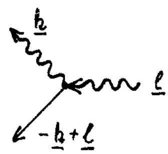


Fig. 5.

As a physically useful result, we have found that the probability of the annihilation of two massless particles is given by the imaginary part of the matrix element of the forward scattering of the two massless particles in second order. This result is of the same structure as in Minkowski space.

6. EMISSION OF A MASSIVE PARTICLE

If one massless particle is going in, the left-hand side of the optical theorem (3.4) decomposes according

$$\sum_{e,s} |\langle \text{in } e^{\phi} s^{\psi} | T^{(1)} | 1_{\mathbf{l}}^{\psi} \text{ in}\rangle|^2 = \sum_{\mathbf{k}} |\langle 5\rangle|^2 + \sum_{\mathbf{q},\mathbf{t}} |\langle 1\text{a}\rangle|^2 + \sum_{\mathbf{q}} |\langle 1\text{b}\rangle|^2 \tag{6.1}$$

See Figure 5 for reference. The squared amplitudes can be interpreted according to

$$\sum_{\mathbf{k}\neq\mathbf{l}} w^{\text{add}}(1_{\mathbf{k}}^{\psi} | 1_{\mathbf{l}}^{\psi})^{(2)} = \sum_{\mathbf{k}} |\langle 5\rangle|^2 \tag{6.2}$$

$$w^{\text{add}}(1_{\mathbf{l}}^{\psi} | 1_{\mathbf{l}}^{\psi})^{(2)} = \sum_{\mathbf{q},\mathbf{t}} |\langle 1\text{a}\rangle|^2 + \sum_{\mathbf{q}} |\langle 1\text{b}\rangle|^2 \tag{6.3}$$

as added-up probabilities, where (6.3) is the probability to find the massless particle undisturbed with the same 3-momentum in the out-region. With (3.4) we get for the optical theorem

$$2 \operatorname{Im}\langle \text{in } 1_{\mathbf{l}}^{\psi} | T^{(2)} | 1_{\mathbf{l}}^{\psi} \text{ in}\rangle = \sum_{\mathbf{k}} w^{\text{add}}(1_{\mathbf{k}}^{\psi} | 1_{\mathbf{l}}^{\psi})^2 \tag{6.4}$$

But not that also the matrix elements $\langle 2\text{a}\rangle$ and $\langle 2\text{b}\rangle$ contribute to this imaginary part:

$$\operatorname{Im}\langle \text{in } 1_{\mathbf{l}}^{\psi} | T^{(2)} | 1_{\mathbf{l}}^{\psi} \text{ in}\rangle = \operatorname{Im}\langle \text{in } 0 | T^{(2)} | 0 \text{ in}\rangle + \operatorname{Im}\langle 6\text{a}\rangle + \operatorname{Im}\langle 6\text{b}\rangle \tag{6.5}$$

Reference is made to Figure 6. Equating (6.4) and (6.5), we obtain with

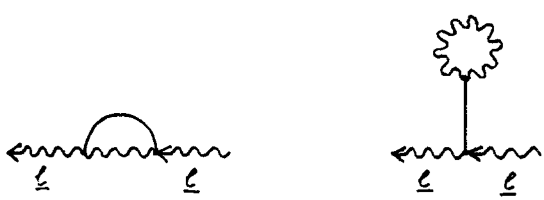


Fig. 6.

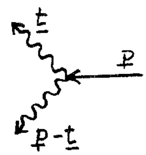


Fig. 7.

(6.3) and (4.2) a more specific result:

$$\sum_{\mathbf{k}\neq\mathbf{l}} w^{\mathrm{add}}(1^{\psi}_{\mathbf{k}} \,|\, 1^{\psi}_{\mathbf{l}})^{(2)} = 2\,\mathrm{Im}\langle 6\mathrm{a}\rangle + 2\,\mathrm{Im}\langle 6\mathrm{b}\rangle \tag{6.6}$$

Because of 3-momentum conservation, the left-hand side may, according to the indicator configuration involved, be interpreted as the total probability for the emission of a massive particle by a massless particle. Now only the diagrams of Figures 6a and 6b are to be worked out. These second-order in–in self-energy transitions of massless particles represent second-order forward scattering in the in–in scheme in a closer sense as compared to (6.4). Because of conformal invariance, their evaluation is similar to the corresponding calculation in the Minkowski space.

7. DECAY OF A MASSIVE PARTICLE

For the case of one massive particle going in, the following diagrams are involved:

$$\sum_{e,s} |\langle \mathrm{in}\; e^{\phi} s^{\psi} |\, T^{(1)} | 1^{\phi}_{\mathbf{p}}\; \mathrm{in}\rangle|^2 = 2\,\mathrm{Im}\langle \mathrm{in}\; 1^{\phi}_{\mathbf{p}} |\, T^{(2)} | 1^{\phi}_{\mathbf{p}}\; \mathrm{in}\rangle$$

$$= \sum_{\mathbf{t}} |\langle 7\rangle|^2 + \sum_{\mathbf{q},\mathbf{t}} |\langle 1\mathrm{a}\rangle|^2 + \sum_{\mathbf{q}} |\langle 1\mathrm{b}\rangle|^2 + |\langle 8\rangle|^2 \tag{7.1}$$

where reference is made to Figures 1, 7, and 8. The connection with a physical interpretation can be established using

$$\sum_{\mathbf{t},\mathbf{q}} w^{\mathrm{add}}(1^{\psi}_{\mathbf{t}} 1^{\psi}_{\mathbf{q}-\mathbf{t}} \,|\, 1^{\phi}_{\mathbf{p}})^{(2)} = \sum_{\mathbf{t}} |\langle 7\rangle|^2 + \sum_{\mathbf{t},\mathbf{q}} |\langle 1\mathrm{a}\rangle|^2 \tag{7.2}$$

and

$$w^{\mathrm{add}}(0^{\psi} \,|\, 1^{\phi}_{\mathbf{p}})^{(2)} = \sum_{\mathbf{q}} |\langle 1\mathrm{b}\rangle|^2 + |\langle 8\rangle|^2 \tag{7.3}$$

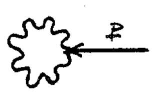


Fig. 8.

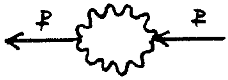


Fig. 9.

The total added-up probability of (7.2) has explicitly been worked out and discussed for particular expansion laws in Audretsch *et al.* (1987). The optical theorem takes the form

$$2\,\mathrm{Im}\langle \mathrm{in}\; 1^{\phi}_{\mathbf{p}} | T^{(2)} | 1^{\phi}_{\mathbf{p}}\; \mathrm{in}\rangle = w^{\mathrm{add}}(0^{\psi} | 1^{\phi}_{\mathbf{p}})^{(2)} + \sum_{\mathbf{t},\mathbf{q}} w^{\mathrm{add}}(1^{\psi}_{\mathbf{t}} 1^{\psi}_{\mathbf{q}-\mathbf{t}} | 1^{\phi}_{\mathbf{p}})^{(2)} \quad (7.4)$$

The imaginary part is based on the diagrams of Figures 1 and 9:

$$\mathrm{Im}\langle \mathrm{in}\; 1^{\phi}_{\mathbf{p}} | T^{(2)} | 1^{\phi}_{\mathbf{p}}\; \mathrm{in}\rangle = \mathrm{Im}\langle \mathrm{in}\; 0 | T^{(2)} | 0\; \mathrm{in}\rangle + \mathrm{Im}\langle 9\rangle \quad (7.5)$$

Again one may like to obtain a more specific expression. To try this, we equate (7.1) and (7.5) and use (4.2):

$$\sum_{\mathbf{t}} |\langle 7\rangle|^2 = 2\,\mathrm{Im}\langle 9\rangle - |\langle 8\rangle|^2 \quad (7.6)$$

Because of

$$w^{\mathrm{add}}(1^{\psi}_{\mathbf{t}} 1^{\psi}_{\mathbf{p}-\mathbf{t}} | 1^{\phi}_{\mathbf{p}})^{(2)} - w^{\mathrm{add}}(1^{\psi}_{\mathbf{t}} 1^{\psi}_{\mathbf{p}-\mathbf{t}} | 0)^{(2)} = |\langle 7\rangle|^2 \quad (7.7)$$

this takes the form

$$\sum_{\mathbf{t}} [w^{\mathrm{add}}(1^{\psi}_{\mathbf{t}} 1^{\psi}_{\mathbf{p}-\mathbf{t}} | 1^{\phi}_{\mathbf{p}})^{(2)} - w^{\mathrm{add}}(1^{\psi}_{\mathbf{t}} 1^{\psi}_{\mathbf{p}-\mathbf{t}} | 0)^{(2)}] = 2\,\mathrm{Im}\langle 9\rangle - |\langle 8\rangle|^2 \quad (7.8)$$

As compared to (7.2), according to (7.7), reference is now solely made to the diagram of Figure 7. Because the probability that the two massless particles of the indicator configuration are produced out of the vacuum is subtracted, we have good reasons to call the left-hand side of (7.8) the total probability of a massive particle to decay into two massless particles. It is a disadvantage of formula (7.8) that in addition to the imaginary part of the self-energy of the massive particle (Figure 9), also the transition amplitude related to Figure 8 appears.

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